

**Five - Year Integrated M.Sc. Examination 2023**

**Subject: Mathematics :**

Paper: [MT-3-5-5 (2016) ] -

(Mathematical Theory of Probability & Statistics)

Full Marks: 40

Time: Three hours

Answer any four of the following six questions.

(Notations carry usual meanings. Calculators may be used)

1. (a) A bag contains 5 red, 7 blue, and 3 green balls. Two balls are drawn at random. What is the probability that they are of different colors? Show your work step by step.
- (b) Three coins are tossed simultaneously. Let  $X$  be the random variable denoting numbers of head appeared. Write down the probability distribution of  $X$ . Hence or otherwise find the probability of getting at least two heads.

5+5

2. (a) A bookstore sells an average of 4 rare books per week. What is the probability that they will sell exactly 2 rare books in a given week? Given  $e^{-4} = 0.01843$ .
- (b) The lifespan of a particular brand of batteries is normally distributed with a mean of 200 hours and a standard deviation of 15 hours. Find the probability that a randomly selected battery will last between 185 and 215 hours. Given  $P(0 < z < 1) = 0.3413$  where  $Z$  follows  $N(0, 1)$  distribution.

5+5

3. (a) Explain with example what do you mean by estimation of parameters? What are the different desirable characteristics of an estimator?
- (b) Consider the following frequency distribution of a random variable  $X$  following Poisson distribution with parameter  $\lambda$ . Estimate the value of unknown parameter  $\lambda$  of the distribution using method of moments and method of maximum likelihood.

No. of misprints	0	1	2	3	4	Total
No. of Pages	109	65	22	3	1	200

5+5

4. (a) From the following frequency distribution of marks in an examination can you estimate the approximate number of students with marks between 32-58? Give reasons for your answer. (Necessary tables will be provided during the examination)

Marks	No. of students
10 - 20	2
20 - 30	5
30 - 40	15
40 - 50	25
50 - 60	12
60 - 70	9
70 - 80	2
Total	70

- (b) A car manufacturer states that their new hybrid car model gets an average of 50 miles per gallon (mpg) on the highway. A consumer group tests 25 cars and finds that the average mileage is 48 mpg with a standard deviation of 3 mpg. Test the manufacturer's claim at a 5% significance level. What is your conclusion? (Given critical value of normal table at 5% level is 1.96)

5+5

5. (a) In the context of testing of hypothesis explain the following terms:  
i) Null Hypothesis, ii) Type-I Type-II errors, iii) Critical region.
- (b) Show that the sample mean ( $\bar{x}$ ) is a consistent estimator of the population mean for the normal distribution.
6. (a) Find an unbiased estimator of the parameter  $p$  in case of  $Bin(n, p)$  distribution on the basis of a random sample of size  $n$  say  $x_1, x_2, \dots, x_n$ .
- (b) Give classical definition of probability and describe its limitations.

5+5